PROPOSAL FOR THE SUMMER MINI-COURSE: AN INTRODUCTION TO HODGE THEORY

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Hodge theory is a powerful tool for the study and classification of algebraic varieties, the Hodge conjecture being one of the main focus points of research in the area. Its influence on algebraic geometry has been greatest in the study of abelian varieties, moduli problems and algebraic cycles. For a smooth projective variety $X \subset \mathbb{P}^N_{\mathbb{C}}$, its de Rham cohomology group $H^k(X, \mathbb{C})$ carries a special decomposition, namely the Hodge decomposition. Based on the Hodge structure on X plus some extra information called the polarization, one can say a lot about X itself. Polarized Hodge structure on X completely determines X when X is a curve, an abelian variety or a K3 surface (the Torelli theorem).

Topics to be covered include (roughly one each week):

(a) The cohomology of smooth projective manifolds: the Hodge and Lefschetz decomposition.
(b) Examples in the curve case. Torelli theorem for curves.
(c) Algebraic cycles and their cohomology class.
(d) Tori associated to cohomology and the Abel-Jacobi maps.
    Possible additional topic include
(e) Griffiths and Clemens' proof of the irrationality of cubic threefolds.

Some standard references are:

(a) Griffiths, Harris: Principles of Algebraic Geometry, Wiley Interscience.
(b) Voisin: Hodge theory and complex algebraic geometry, Volume I and II, Cambridge University Press.
(c) Calson, Muller-Stach, Peters: Period mappings and Period Domains. Cambridge University Press.